Rational Bubbles and Economic Crises: 
A Quantitative Analysis*

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Abstract

We extend the Bewley-Aiyagari-Huggett model by incorporating an incomplete stock market and a persistent income process. In this quantitative general equilibrium framework, non-fundamental asset values are both large and desirable for realistic parameter values. However, if expectations shift from one equilibrium to another, some markets may crash as others soar. In the presence of nominal assets and contracts, such movements can be highly detrimental. Our analysis is consistent with the view that some of the world’s large recessions were caused by an avoidable failure of monetary and fiscal policy to prevent deflation in the aftermath of bursting asset price bubbles.

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1 Introduction

It is frequently suggested that bursting asset price bubbles precipitated the two major worldwide recessions during the last century as well as many other economic crises (e.g., Reinhart and Rogoff, 2009, Chapter 13). For the economics profession, this is an awkward hypothesis. Asset price bubbles play no role in the quantitative models that underpin policy-making in finance ministries and central banks. Instead, in responding to the recent global recession, we have relied on models in which the recession is caused by large shocks to preferences or technology.

In this paper, we develop a medium-sized quantitative macroeconomic model with a central role for rational asset price bubbles and their cousins, rational Ponzi-values. The model implies that large non-fundamental asset values are the rule rather than the exception, and it allows self-fulfilling changes in beliefs to propel such values from one asset class to another. In particular, a stock-market crash can coincide with a surge in bond prices. If public debt policy is passive, the outcome then depends on whether the debt is real or nominal. If public debt is real, the real interest rate drops. If public debt is nominal, the price level drops. In the latter case, which is most relevant in practice, nominal wage stickiness may generate several years of unemployment. Expansionary monetary policy in the form of temporary nominal interest-rate adjustments can only dampen the detrimental effects to a limited extent. On the other hand, fiscal policy in the form of a one-time expansion of public debt can prevent both unemployment and excessively low real interest rates.

The theory of rational asset price bubbles is controversial. It enjoyed a decade of popularity in the 1980s (Blanchard and Fischer, 1989, Ch. 5.) However, the influential empirical work of Abel et al. (1989) concluded that the return to capital investment had been robustly exceeding the rate of growth for long periods of time, thus contradicting the central assumption required for the existence of rational bubbles: $r < g$. Many theorists also became convinced that the theory was practically irrelevant. The degree of market incompleteness that would be required to produce $r < g$ was considered too great (e.g., Santos and Woodford, 1997), and numerical experiments suggested that rational bubbles wouldn’t much matter for optimal policy anyway (Kehoe, Levine, Woodford, 1992). A decade later, LeRoy (2004, page 801) reluctantly concluded: “Within the neoclassical paradigm there is no obvious way to derail the chain of reasoning that excludes bubbles”. Hence, while academic interest in bubbles did not entirely subside, the focus turned to irrational bubbles\[1\]

But the evidence against rational bubbles is weaker than it might seem. The real return to holding short-maturity government debt has been lower than the growth rate

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\[1\]See Scheinkman (2014) for a recent survey of the literature on irrational bubbles.
in many countries and for long periods of time. In other words, the return to such reasonably safe assets has historically been well below the growth rate. Moreover, Abel et al. (1989) effectively measure average returns to all investment, not the marginal returns that accrue to portfolio investors. In the presence of scale economies or financial frictions, the latter can be considerably smaller than the former (e.g., Woodford, 1990; Farhi and Tirole, 2012). Finally, when Geerolf (2013) revisits the analysis of Abel et al. (1989) with better data, he finds that the average real return to capital investment is considerably smaller than previously thought, and quite possibly below the rate of growth. In summary, we cannot discard the theory of rational bubbles on empirical grounds.

The paper makes three main contributions. First, we demonstrate that large non-fundamental asset values, in the form of rational bubbles and Ponzi-values, are theoretically likely. Specifically, we extend the dynastic general equilibrium model of incomplete markets developed by Bewley (1980, 1983), İmrohoroğlu (1989), Huggett (1993), and Aiyagari (1994) in two directions. First, we add to the model a stock market, in which agents may trade claims to many existing entrepreneurial ideas, but not to future ideas. Instead of trading frictions caused by overlapping generations of people (OLG) – which give rise to bubbles in Samuelson (1958), Diamond (1965) and Tirole (1985) – we consider frictions caused by overlapping generations of assets (OLGA). Entrepreneurial rents increase asset demand and the stock market increases asset supply compared to the baseline rent–free model. Besides added realism, the presence of a stock market allows us to study bubble movements from one asset class to another, thus resuscitating the theory of crashes under rational expectations due to Tirole (1985). Second, we introduce an uninsurable labor income process that is persistent enough to emulate the life-cycle savings motive, creating additional demand for assets. Under reasonable parameters, these two sources of uncertainty together with realistic borrowing constraints suffice to generate a real interest rate below the rate of growth, even after accounting for a large non-fundamental asset price component. That is, our two extensions suffice to resolve the “low risk-free rate” puzzle without assuming an implausibly incomplete asset market.

2In the US since 1950, the average short-run real interest rate on government debt is around 1 percent (from 1985 it is 1.5 percent), whereas the average rate of productivity growth is 2 percent.

3As a case in point, consider Facebook, an internet company started by novices in 2003. When it became publicly traded in 2010, the company was worth around 40 billion US dollars. By contrast, in a complete asset market, well diversified investors would already have held financial claims on Facebook, and any other venture that the founder Mark Zuckerberg may have initiated, long before 2003.

4The idea of introducing life-cycle savings motives into the dynastic framework is not new. For a quantitative investigation along these lines, see Castañeda, Díaz-Giménez and Ríos-Rull (2003).

5The hypothesis that precautionary saving could in principle account for the $r < g$ puzzle has been pursued for several decades; see in particular Aiyagari and Gertler (1991) and Huggett (1993). However, a typical quantitative result in this early literature is that one can shave little more than a percentage point off the complete markets interest rate, $g + r_s$ (where $r_s$ is the subjective discount rate). More
A second contribution is conceptual. In our model with multiple private assets as well as government bonds, it becomes clear that aggregate non-fundamental values are composed of both bubbles and Ponzi-values, and that their proportions are not rigidly determined. Thus, even as aggregate non-fundamental values remain constant, bubbles could migrate between different assets, or they could transform from bubbles to Ponzi-values and vice versa\[6\]

Third, we emphasize the importance of nominal assets. Public debt is typically denominated in terms of money rather than in terms of output. Therefore, an unchecked increase in demand for public debt will drive down the price level rather than merely reducing the real rate of interest\[7\]. With nominal price and wage flexibility, this is not a problem. A bursting bubble on private assets simply raises the price of existing public debt so as to keep the interest rate constant, and all other prices and wages adjust to the new price level. But under nominal wage rigidity and passive public policy, the real value of fixed nominal wages goes up. According to the model, there are then two main ways to avoid large welfare losses. The first way involves maintained real interest rates, with public debt growing to accommodate the non-fundamental value that migrates from the private sector. The second way involves a maintained level of public debt and a firm commitment to permanently lower real interest rates.

Of course, at least since the Great Depression, generations of economists have called for deficit spending to mitigate large recessions. However, while supporting their prescription, we argue that a crucial part of their diagnosis is misleading. Keynes and his followers argue that depression arises because current aggregate demand is too small at prevailing prices. Faced with such shortfall in demand for current consumption relative to future consumption, authorities ought to increase their spending to plug the gap. By contrast, we argue that depression is caused by increased demand for one type of savings vehicle, government debt, relative to others, such as stocks or property. Thus, when the demand for debt goes up, authorities ought to increase the supply of bonds so as to prevent an undesired increase in real wages due to nominal rigidities.

recently, in a model without durable assets, but with aggregate shocks and maximally tight borrowing limits, Krusell, Mukoyama, and Smith (2013) are able to reproduce the observed lower risk-free rate. However, this result is vulnerable to the critique of Santos and Woodford (1997) that asset markets are assumed to be unrealistically incomplete.

\[6\] Other work has touched on the close relationship between bubbles and debt; see in particular Hellwig and Lorenzoni (2009).

\[7\] Kocherlakota (2011), in his qualitative discussion of how bursting bubbles can affect the government bond market, only considers the case of real public debt.
2 Concepts and Related Literature

If all investors have rational expectations, an asset’s non-fundamental value can only be positive if the asset’s duration is indefinite. Thus, our model has an indefinite (infinite) horizon.

2.1 Ponzi-values and bubbles

We consider two types of non-fundamental values: Ponzi-values and bubbles. While these definitions will be quite standard, we find it useful to express them in the context and notation of the paper. To fix ideas, think about an infinitely small issuer of a financial asset, living in an economy that grows at rate $g$ and with an equilibrium real interest rate of $r$.

Suppose first that the financial asset is real debt. We define the Ponzi-value of a debt contract as the difference between the (market) price that lenders are willing to pay for it and the net present value of the sacrifices that the borrower must make in order to fulfill the contract. The narrowest definition of a Ponzi-scheme would insist that the issuer makes no sacrifice at all, but is able to roll over the debt, including interest payments, forever.

If $r < g$, the issuer’s sacrifices are zero along a balanced growth path, and the aggregate Ponzi-value of an economy equals the market value of those debts that are expected to be rolled over indefinitely. When the agreed interest rate is constant at $r$, the Ponzi-value of a real debt thus equals its face-value.

When $r < g$, issuers of indefinite debts potentially benefit immensely. For example, someone who issues new debt at the rate $g$ can consume a fraction $g - r$ of the accumulated debt every period. In modern countries, most agents are legally precluded from expanding net debts indefinitely. Gross debts should be matched by corresponding assets, and agents who are unable ever to serve their debt are considered bankrupt.

Thus, the value of the issuer’s sacrifice equals the value to the debt holder. However, central governments usually do have the legal right to expand the public debt indefinitely, and they usually exploit it. Few countries target a long-run debt-to-GDP ratio of zero. Along a balanced growth path, the value of government debt is wholly non-fundamental if $r < g$. If $r > g$, the borrower must make some sacrifice to service the debt, whose value is thus only partially non-fundamental.

Suppose next that the financial asset is equity. Specifically, suppose the asset issuer owns a productive resource. The resource has a constant survival probability $\sigma$, and every period $t$ that it survives it yields a total dividend $d_t = (1 + g)^t d_0$. Finally, let

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8Bankruptcy laws that preclude Ponzi-schemes are particularly stringent for financial intermediaries. There are often related laws that constrain the net indebtedness of local governments.
us assume that $\sigma(1 + g) < 1 + r$, and that the idiosyncratic risk associated with the stock’s survival can be perfectly diversified. Since investors then discount this future dividend stream at rate $r$, the fundamental value of the shares is

$$f_t = \frac{\sigma d_{t+1}}{1 + r} + \frac{\sigma^2(1 + g)d_{t+1}}{(1 + r)^2} + \ldots$$

$$= \frac{\sigma d_{t+1}}{1 + r - \sigma(1 + g)}, \quad (1)$$

where the last equality is a consequence of our assumption $\sigma(1 + g) < 1 + r$ (otherwise, the fundamental value would be infinite). The price of a share need not be equal to the fundamental value, however. Rather, the price $p_t$ needs to satisfy the first-order difference equation

$$p_t = \frac{\sigma(p_{t+1} + d_{t+1})}{1 + r}, \quad (2)$$

which, when $\sigma(1 + g) < 1 + r$, has the general solution

$$p_t = f_t + b_t, \quad (3)$$

with $\{b_t\}$ satisfying

$$b_t = \frac{b_{t+1}}{1 + r}.$$

The variable $b_t$ is the stock’s rational bubble. In other words, a rational bubble on a company’s shares corresponds to the net present value of owning the share “at infinity.”

In expectation, a rational bubble grows at the real rate of interest. This means that the bubble on a surviving stock must grow at the rate $(1 + r)/\sigma - 1$. Thus, if the bubble process is the same across generations of firms, the bubble is larger on older stocks, although eventually these stocks are so rare as to constitute a small fraction of the aggregate bubble.

Notice that we could in principle have a bubble on a stock that yields zero fundamental value. By extension, there could be rational bubbles on all kinds of unproductive assets. Rare stamps, paintings, and other expensive collectors’ items are relevant examples.

### 2.2 Other literature on non-fundamental asset values

Most of the recent work on rational asset price bubbles has considered the role of financial frictions. There are two central ideas, occurring sometimes separately and

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9The case of currency is special, as it might be seen either as an infinite debt that entails exactly zero interest (indeed, the monetary base is usually considered to be a component of public debt) or as a bubble. However, when currency yields transactions services, it cannot also be a bubble; see Tirole (1985).
sometimes jointly: First, bubbles on an entrepreneur’s assets add to net wealth and thus help to support investment. Second, bubbles have higher liquidity than other assets and thus facilitate investment when financial constraints are particularly tight. Contributions to this literature include Woodford (1990), Kiyotaki and Moore (2002, 2003), Farhi and Tirole (2012), Martín and Ventura (2012), and Wang and Wen (2012). We refer to these papers for a more detailed discussion and further references. Note that this literature implies that the demand for saving by entrepreneurs or firms is larger than we assume and that their supply of assets is more limited than we assume. Hence, if we were to take financial frictions into consideration, it would be even easier to support rational bubbles.

A notable difference between our approach and the financial frictions approach concerns the impact of a broad stock market crash. The financial frictions approach implies that there will be a recession due to the reduction of entrepreneurs’ net wealth. By contrast, our approach suggests that there will be a major recession only if the bubble moves into the bond market and thereby affects the price level. If, for example, the bubble moves into the residential housing market, economic activity may be largely unaffected. In that way, our model allows a simple interpretation of the events in the decade 2000-2010: The dotcom crash had a minor impact on output because the bubble migrated to property markets. The joint crashes of property and stock markets in 2007-8 had a major impact on output because the bubble migrated to government bond markets (where it morphed into Ponzi-value).

3 The Model

Time is discrete, and the horizon is infinite. Period \( t = 0 \) refers to the current period. Periods \( t = -\infty, \ldots, -1 \) comprise the history and determine the “initial conditions” that characterize period 0. Periods \( t = 1, \ldots, \infty \) comprise the future. There is a continuum of infinitely lived agents distributed along the unit interval.

Preferences: Agents consume a homogeneous final good and have identical preferences. Their utility function is of the form

\[
U = E \sum_{t=0}^{\infty} \beta^t u(c_t),
\]

where \( c_t \) is consumption in period \( t \), and \( \beta \in (0, 1) \) is the subjective discount factor. We assume that the felicity function \( u \) takes the CRRA form

\[
u(c) = \frac{c^{1-\mu}}{1-\mu},
\]
Inalienable endowments: (i) Labor. Each period, an agent is endowed with one unit of labor. Labor has no opportunity cost, and is thus supplied inelastically. The variable $\varepsilon_{i,t}$ indicates whether a agent $i$ is productive ($\varepsilon_{i,t} = 1$) or not ($\varepsilon_{i,t} = 0$). We think of the unproductive state as retirement and the productive state as working-age. Age follows a first order Markov process. (ii) Ideas. The variable $\varepsilon_{e,t}$ denotes whether an agent $i$ is endowed with an entrepreneurial idea at date $t$ ($\varepsilon_{e,1} = 1$) or not ($\varepsilon_{e,0} = 0$). Ideas survives to the next period with probability $\sigma < 1$.

Technology: (i) Final goods. The production of final goods is conducted by competitive producers, and requires neither ideas nor labor. Instead, it requires a continuum of intermediate goods. The final goods production function is

$$Y_t = \left( \int_0^A (y_{n,t})^\eta \, dn \right)^{1/\eta},$$

where $y_{n,t}$ is an intermediate input, and where $0 < \eta \leq 1$.

(ii) Intermediate goods. The intermediate goods sector consist of a continuum $A$ of monopolies. Each monopoly $n$ requires a unique idea that allows it to produce a specific intermediate good by combining physical capital $k$ and labor $h$. The capital is in the form of final goods. The intermediate goods production technology is

$$y_{n,t} = (k_{n,t})^\alpha (\lambda_t h_{n,t})^{1-\alpha},$$

where labor productivity $\lambda_t = (1 + g)^t$ grows at an exogenous rate $g$. If an idea becomes obsolete, the firm ceases to exist.

Firms: All new firms are initially private companies, and the agent owning the idea bears all the risk. Each period, each entrepreneur is enabled with probability $\sigma_o$ to (costlessly) issue equity in their firm and transform it into a public company. Let $\varepsilon_{o,i}$ denote whether agent $i$ received an opportunity to make an IPO ($\varepsilon_{o,i} = 1$) or not ($\varepsilon_{o,i} = 0$). Let $A_1$ denote the continuum of private companies and $A_2$ the continuum of public companies, where $A = A_1 + A_2$.

Assets and asset holdings: There are three kinds of tradable assets: (i) Capital corresponds to the renting out of final goods to be used as an input in production. Let $r_t^k$ be the market rate of return to capital at date $t$, and let $\delta$ be the capital depreciation rate. (ii) Stocks are financial claims on the profit $d_n$ of an intermediate goods producer $n$, to be described below. (iii) One-period government bonds constitute a promise by the authorities of a fixed nominal repayment $1 + r_t^m$. That is, the interest on a bond, $r_t^m$ is expressed in terms of bonds rather than final goods. Let $M_t$ denote the volume of government bonds issued at date $t$.

Let $m_{i,t}$ and $k_{i,t}$ denote agent $i$’s holding of bonds and physical capital respectively, and let $a_{i,n,t}$ denote the agent’s holding of stocks in firm $n$. 

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Trade: There are frictionless markets for final goods, intermediate goods, labor, capital, and stocks in public firms. We express all prices and payments (except the interest on government bonds) in terms of final goods. Specifically, the price of capital is always 1 (since the final good is our numeraire), $w_t$ is the wage at date $t$ for a productive worker, $p_{n,t}^y$ is the price of intermediate good $n$, and $p_t^m$ is the price of government bonds.

For simplicity, we shall assume that investors do not keep track of companies’ birth. At most, they remember the previous period’s market price. Thus, to the extent that companies’ price depend on the company’s history, the dependence only involves price movements since the date they where listed. Henceforth, let $p_{j,t}$ denote the price at date $t$ of a public company that was listed at date $j$.

Government: The government sets the nominal interest rate $r_t^m$ and the growth rate $g_t^m$ of the stock of nominal government debt $M_t$. The government imposes a proportional pension tax on labor income of working-age agents, $\tau_t^p$, pays lump-sum pensions $\theta_t$ to retirees, and makes lump-sum payouts $\tau$ to all agents.

Within-period timing: (i) Production takes place and income is distributed. (ii) A fraction $\sigma_0$ of the hitherto private entrepreneurs get the opportunity to take their firm public. (iii) Agents consume and save. (iv) New ideas are realized. (v) A fraction $1 - \sigma$ of all ideas become obsolete and the associated intermediate goods firms are destroyed. Thus, the ratio of public to private firms is $A_2/A_1 = \sigma_0/(1 - \sigma)$ and $(1/\sigma - 1 + \sigma_0) A_1$ new ideas are realized each period. The total supply of equity at the end of a period is $\bar{A}_2 = A_2/\sigma$.

An agent’s state and expectations: From now on, let us suppress agent indices $i$. Moreover, let us anticipate that agents (since they are risk-averse) will not want to bear unnecessary idiosyncratic risk. Since markets are frictionless, it is optimal to hold perfectly diversified portfolios of tradable stocks. Let $a_{j,t}$ denote an agent’s holdings at date $t$ of a diversified portfolio of public companies that became first listed at date $j$, and let $a_t = \{a_{j,t}\}_{j=-\infty}^{t-1}$ be the agent’s holding of the market portfolio of all tradable stocks. To ease notation, and without significant loss of insight, we henceforth assume that agents hold the market portfolio of publicly traded assets. Let $p_t$ denote the price of the market portfolio.

Let $\epsilon_t = (\epsilon_t^x, \epsilon_t^e, \epsilon_t^o)$. The current state of an agent can then be expressed as $s_0 = (a_0, m_0, k_0, \epsilon_0) \in S$, where $S = \mathbb{R}_+^3 \times \{0, 1\}^3$. Let $\mathcal{R}$ denote the Borel sets that are subsets of $\mathbb{R}_+$, and denote $S = \mathcal{R}^3 \times \{0, 1\}^3$. (This set includes the set of all possible relevant states for the economy.)

We assume that age is independent of entrepreneurship and IPO opportunities. Moreover, we assume that (i) an agent can at most have one currently relevant idea, (ii) an entrepreneur who receives an opportunity to make an IPO can not receive a new idea in the same period, and (iii) a newborn firm can not enter the stock market. Let $\Gamma$
denote the Markov transition matrix that describes how $\varepsilon_t$ evolves over time.

We can then describe expectations about the future as follows. Let $\varepsilon^t = \{\varepsilon_0, \ldots, \varepsilon_t\}$ denote the partial sequence from period 0 up to period $t$ and let $\mathcal{E}^t$, denote the corresponding set of all possible such sequences. Define probability measures $\gamma^t(s_0, \cdot) : \mathcal{E}^t \to [0,1]$, $t = 0, 1, \ldots$, where, for example, $\gamma^t(s_0, \varepsilon^t)$ is the probability of history $\varepsilon^t$ given an agent’s initial state $s_0$.

3.1 Analysis

Production: Since markets are frictionless, and there are no irreversibilities, firms will solve their static profit maximization problem each period.

Final goods producers are competitive solve the problem

$$\max_y Y_t - \int_0^A p^y_{n,t} y_{n,t} dn,$$

where $p_{n,t}$ is the price of intermediate good $n$. From the first-order condition, we define the demand

$$p^y_{n,t}(y_{n,t}) \equiv \left( \frac{y_{n,t}}{Y_t} \right)^{\eta^{-1}}.$$

Each intermediate goods producer $n$ maximizes profits

$$d_{n,t} = p^y_{n,t}(y_{n,t}) y_{n,t} - w_t h_{n,t} - r^k_{t} k_{n,t}$$

by renting capital $k_{n,t}$ at price $r^k_{t}$ and hiring labor $h_{n,t}$ at price $w_t$, subject to (5).

Since each firm solves the same problem, and that problem has a unique solution, we can write $y_{n,t} = y_t$. Thus in any equilibrium, $Y_t = A^{1/\eta} y_t$ and $p^y_{n,t} = p^y_{t} = A^{1/\eta - 1}$.

From the first-order conditions,

$$r^k_t = \eta A^{1/\eta - 1} k_t^\alpha (\lambda_t h_t)^{(1-\alpha)} k_t^{-1},$$

$$w_t = \eta (1-\alpha) A^{1/\eta - 1} k_t^\alpha (\lambda_t h_t)^{(1-\alpha)} h_t^{-1},$$

which implies that profits are given by

$$d_t = A^{1/\eta - 1} k_t^\alpha (\lambda_t h_{n,t})^{(1-\alpha)} (1-\eta).$$

Aggregate labor demand is $H_t = Ah_t$, and aggregate demand for physical capital is $K_{t+1} = Ak_{t+1}$.

Consumption: In the current period, conditional on the current state $s_0$, each agent plans consumption and savings for each possible future sequence $\varepsilon^t$. Let $\phi_t : \mathcal{E}^t \to \mathbb{R}^3_+$, $t = 0, 1, \ldots$, describe the savings plan, where $\phi_{a,t}(\varepsilon^t; s_0)$ denotes the value for $a_{t+1}$ that
is chosen in period $t$ if the history up to $t$ is $\epsilon_t^t$. Similarly $\phi_{m,t}(\epsilon_t^t; s_0)$ denotes the value for $m_{t+1}$, and $\phi_{k,t}(\epsilon_t^t; s_0)$ denotes the value for $k_{t+1}$. For future reference, let the set of savings vectors be denoted $X \subset \mathbb{R}^3_+$. Let $c_t : \mathcal{E}^t \to \mathbb{R}_+$ describe the associated plan for consumption.

At the beginning of period $t$, a fraction $\sigma$ of the public firms alive at $t-1$ will have died; the remaining will yield a dividend $d_t$. Labor income is $y_t = (1 - \tau - \tau^p) (w_t \epsilon_t^t + (1 - \epsilon_t^t) \theta_t)$. An agent’s budget constraint is therefore

$$c_t(\epsilon_t^t; s_0) = y_t + \tau_t^m + (p_t + d_t) \sigma a_t - p_t a_{t+1}$$
$$+ p_t^m ((1 + r_t^m) m_t - m_{t+1})$$
$$+ (r_t + 1 - \delta) k_t - k_{t+1} + d_t \epsilon_t^t + p_t \epsilon_t^0. \quad (9)$$

The agent’s problem is thus to choose a feasible plan $\phi_t(\epsilon_t^t; s_0)$ to maximize expected discounted lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u \left( c_t(\epsilon_t^t; s_0) \right) \gamma^t(s_0, \epsilon_t^t) \quad (10)$$

subject to (9), the transition matrix $\Gamma$, anticipated sequences of prices, and the initial state $s_0$.

This economy features exogenous growth of productivity. Moreover, government policy will be determining a growing supply of bonds. To solve the model, it is therefore convenient to normalize real variables by the level of productivity and nominal variables by the quantity of nominal bonds. A tilde denotes that the variable is detrended by productivity, and a bar denotes that it is de-trended by nominal debt. For example, $\tilde{c}_t \equiv \frac{c_t}{\lambda_t}$, $\tilde{p}_t^m \equiv \frac{p_t^m M_t}{\lambda_t}$, and $\bar{m}_t \equiv \frac{m_t}{\bar{M}_t}$.

**Markets:** Market clearing in financial markets is given by

$$\bar{M}_0 = \int_X \bar{m}_0 d\kappa, \quad t = 0,$$  \quad (11)

$$\sigma^{t-j}(1 - \sigma) \tilde{A}_2 = \int_X \tilde{a}_0 d\kappa(s_0), \quad t = 0,$$  \quad (12)

$$\bar{K}_0 = \int_X \bar{k}_0 d\kappa, \quad t = 0,$$  \quad (13)

$$\bar{M}_t = \int_X \sum_{\epsilon_t^{t-1} \in \mathcal{E}_t^{t-1}} \phi_{m,t-1}(\epsilon_t^{t-1}; s_0) \gamma^{t-1}(s_0, \epsilon_t^{t-1}) d\kappa, \quad \forall \ t > 0,$$  \quad (14)

$$\sigma^{t-j}(1 - \sigma) \tilde{A}_2 = \int_X \sum_{\epsilon_t^{t-1} \in \mathcal{E}_t^{t-1}} \phi_{j,t-1}(\epsilon_t^{t-1}; s_0) \gamma^{t-1}(s_0, \epsilon_t^{t-1}) d\kappa, \quad \forall \ t > 0, \forall \ j \geq -\infty,$$  \quad (15)

$$\bar{K}_t = \int_X \sum_{\epsilon_t^{t-1} \in \mathcal{E}_t^{t-1}} \phi_{k,t-1}(\epsilon_t^{t-1}; s_0) \gamma^{t-1}(s_0, \epsilon_t^{t-1}) d\kappa, \quad \forall \ t > 0.$$  \quad (16)
Similarly, goods market clearing implies that
\[ \dot{Y}_t = \int_X \sum_{e' \in \mathcal{E}} c_t(e'; s_0) \gamma'(s_0, e') \, d\mathbf{x} + \mathcal{G}_t + (1 + g) \bar{K}_{t+1} - (1 - \delta) \bar{K}_t. \]

Finally, labor market clearing implies that \( H = 1 \).

**Government:** To facilitate accounting, and without loss of generality of our results, we depict the government as running three separate budgets. Government expenditure is financed through a labor income tax
\[ \bar{G}_t = \tau \bar{w}_t H, \quad (17) \]
public pensions is financed through a pension-tax
\[ \int_X \sum_{e' \in \mathcal{E}, e'_t=1} (1 - \tau^p) \bar{d}_t \gamma'(s_0, e') \, d\mathbf{x} = \tau^p \bar{w}_t H, \quad (18) \]
and the lump-sum transfer is financed through the growth of nominal public debt net of interest payments
\[ \tau^m_t = (g^m_t - r^m_t) \bar{p}^m_t \bar{M}_t, \quad (19) \]

**Equilibrium:** An equilibrium is sequences of prices \( \{ \bar{p}_t, \bar{p}^y_t, \bar{p}^k_t, \bar{r}_t^1, \bar{w}_t \}_{t=0}^{\infty} \), dividends \( \{ \bar{d}_t \}_{t=0}^{\infty} \), transfers \( \{ \bar{t}^m_t \}_{t=0}^{\infty} \), pensions \( \{ \bar{\theta}_t \}_{t=0}^{\infty} \), government expenditure \( \{ \mathcal{G}_t \}_{t=0}^{\infty} \), aggregate capital \( \{ \bar{K}_t \}_{t=0}^{\infty} \), debt growth rates \( \{ \bar{g}^m_t \}_{t=0}^{\infty} \), nominal interest rates \( \{ r^m_t \}_{t=0}^{\infty} \), tax rates \( \{ \tau, \tau^p \} \), decisions \( \{ c_t(e'; s_0), \phi_t(e'; s_0) \}_{t=0}^{\infty} \) for all \( s_0 \in S \) and for all \( e' \in \mathcal{E}' \), together with probability measures \( \{ \gamma'(s_0, z) \}_{t=0}^{\infty} \) for all \( s_0 \in S \) and for all \( z \in \mathcal{E}' \), and a measure \( \kappa(x) \) for all \( x \in X \) describing the initial distribution, such that (i) the decision rules solve the agents’ problem given prices and the initial state \( s_0 \), (ii) factor prices are given by (6)-(7), and \( p^y_t = A^{1/\eta - 1} \), (iii) dividends are given by (8), (iv) all markets clear, (v) the government budget constraints (17)-(19) are satisfied, and (vi) the measure \( \gamma'(s_0, e') \) is consistent with the transition matrix \( \Gamma \).

**Solution:** The key to solving the model are the no-arbitrage conditions. Let the variables \( \bar{R}^a_t = \sigma(\bar{p}_{t+1} + \bar{d}) / \bar{p}_t \), \( \bar{R}^k_t = (\bar{r}^k_{t+1} + 1 - \delta) / (1 + g) \), and \( \bar{R}^m_t = \bar{p}^m_{t+1} (1 + r^m_t) / [\bar{p}^m_t (1 + g^m_t)] \) denote the growth-adjusted return to holding equity, physical capital and bonds respectively. If nominal bonds are valued, then no-arbitrage implies that \( \bar{R}^a = \bar{R}^k = \bar{R}^m = (1 + r^m) / (1 + g^m) \). Thus, the real return to holding equity, physical capital and bonds respectively is
\[ R^a = R^k = R^m = (1 + g)^{1 + r^m} / (1 + g^m). \]
Note that the steady-state real interest rate is given by $g + r^m - g^m$. Hence, monetary and fiscal authorities jointly determine the real interest rate.

Let $x_t \equiv \bar{p}^m_t \bar{M}_t$ denote the aggregate demand for government debt, and let $g^x_t$ denote the percentage change in $x_t$ between $t - 1$ and $t$. Then,

$$g^x_t = \frac{(1 + g^m_t)}{(1 + \pi_t)(1 + g)} - 1,$$

where $\pi_t \equiv p^m_{t-1}/p^m_t - 1$ denotes inflation. Thus, inflation is

$$\pi_t = \frac{(1 + g^m_t)}{(1 + g^x_t)(1 + g)} - 1 
\approx g^m_t - g - g^x_t. \quad (20)$$

Along a balanced growth path, $g^x_t = 0$ and inflation thus satisfies the quantity theory,

$$\pi \approx g^m - g. \quad (21)$$

However, in the short run (along a transition path), changes in the demand for government debt will cause inflation to deviate from the quantity theory.

4 The Magnitude of the Non-fundamental

Let us now compute the aggregate non-fundamental value in a calibrated steady state of our model. We aim to fit macro-figures for the United States in recent times, so we will be in a regime with $r < g$. More specifically, we think of the model as representing average US fundamentals during the last two decades.

Instead of guessing the location of non-fundamental asset values, we assume for now that the whole non-fundamental value takes the form of (Ponzi) public debt, which is thus the residual variable to be determined.

Since agents in our model experience heterogeneous histories, wealth and income distribution are important targets for our calibration. In order to produce a suitable fraction of rich agents in our model, without introducing many different wage levels, our assumption about entrepreneurial incomes is central. Here, we set the fraction of the population that has received an idea that is still in use to 1.5 percent ($A_1 = 0.015$). As will become clear, this parameter choice contributes strongly to the model’s ability to match the observed Gini-coefficient for wealth, which will be 0.82 – as it is in US data (see Díaz–Giménez, Glover and Ríos-Rull, 2011).

Regarding all remaining parameters we either use standard values from the literature or calibrate the parameters to match long-run averages in US data. The model period is one year. The discount factor $\beta$ is 0.97, and the coefficient of relative risk
aversion $\mu$ is 3. The price mark-up is set to 20 percent ($\eta = 5/6$), which is in the mid-range of estimates of US markups. The capital share in production, $\alpha = 0.196$, is set to match a labor share in income of $2/3$. The depreciation rate, $\delta = 0.06$, is set to match an investment to GDP ratio of 0.17. The exogenous growth rate of productivity, $g$, is set to 2 percent, to match the average growth rate in real GDP/hour between 1950 and 2010.

We set the survival probability for intermediate goods producers, $\sigma$, to 0.98 so that the value of new firms is 2 percent of total stock market value, matching the long-run average reported by Jovanovic and Rosseau (2005). The fraction $\sigma_o$ of entrepreneurs who get the opportunity to make an IPO is set so that the share of private firms is 90.5 percent; $\sigma_o = 0.00215$. These choices help us match the capital-output ratio as well as the stock-market-to-GDP ratio. According to Caselli and Feyrer (2007), the US capital-output ratio is 2.19 and according to data from the World Bank the stock market to GDP ratio in the United States has in the last twenty years varied between 0.6 and 1.6, with an average of 1.09. In the model, these are 2.22 and 1.05 respectively.

We interpret working-age and retirement as corresponding to ages 20-64 and 65-79 respectively, to match a retirement age of 65 and an expected longevity of 79 years. Hence, we set the transition probabilities between working-age and retirement so that agents spend on average 45 years working followed by 15 years in retirement. The pay-roll tax is set to 8.5 percent, so that Social Security payments are 4.2 percent of GDP, which is what Wallenius (2013) reports for the United States. The labor income tax, $\tau$, is set to 26 percent, implying a government expenditure of 19 percent of GDP.

Finally, regarding the policy parameters, recall that only the difference between $r^m$ and $g^m$ matters. Between 1950 and 2010, US CPI inflation has been on average 3.8 percent and the nominal interest on treasury bills has on average been 4.8 percent. Thus, the average real interest rate has been 1 percent since 1950. However, since the change in monetary policy in the early 1980’s the real interest rates has been substantially higher for most of the time, and we have therefore chosen to match the post 1985 average of 1.5 percent. Specifically, we implement an nominal interest rate of 3.5 percent and a money growth rate of 4 percent, implying an inflation of 2 percent in the stationary equilibrium.

These policies imply a government debt to GDP ratio of 120 percent, which is considerably larger than the actual figure of US public debt, especially when we take into account the debt that is held by foreigners. Thus, we conclude that the model admits sizable bubbles on private assets in addition to the Ponzi-value contained in the public

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10 See for example Basu and Fernald (1997).
11 Interpreting private firms as “small” firms, this is the number reported by Bartelsman et al. (2004) for industrialized countries.
12 The World Bank data refer to Stock Market Capitalization to GDP for United States, series id DDDM01USA156NWDB, World Bank, Global Financial Development.
To assess the quantitative importance of our stock market assumptions, we also solve the model for two alternative scenarios. (i) Assume that all ideas die with certainty after a single period \((\sigma = 0)\). This is essentially the model economy in Aiyagari and McGrattan (1998), albeit with a slightly different production structure. In this case, there cannot be stock market bubbles. Moreover, non-fundamental values are smaller (96 percent of GDP rather than 120), and wealth inequality less pronounced (Gini is 0.57 instead of 0.82). (ii) Assume that ideas last forever \((\sigma = 1)\). This assumption corresponds quite closely to that of Santos and Woodford (1997), where any new assets are owned by holders of old assets. In this case, the model can no longer rationalize the interest rate of 1.5 percent. Instead, keeping parameters constant, the smallest equilibrium real interest rate is 2.88 percent.

Compared to these two regimes, our parameter choice of \(\sigma = 0.98\) is not only more realistic; it also yields a better fit to the data.

5 A Stock Market Crash

The equilibrium computed above is not unique. In fact, for any \(r < g\) there exists a continuum of stationary equilibria. Loosely speaking, these equilibria can be indexed by the real value of debt \((\tilde{p}_m^p \tilde{M}_t)\), the associated lump-sum transfers \((\tilde{\tau}_m^t)\), the price of new public firms \(\tilde{p}_{t,t}\) and the asset distribution \(\kappa\). For example, consider our benchmark calibration where policy is such that the real interest rate is 1.5 percent. Then one equilibrium (the one studied above) entails no price bubbles on stocks and a public debt to GDP ratio of 1.2. But there also exist equilibria for any value of public debt in the range 0 – 120 percent of GDP: The lower the public debt, the larger is the price of new public firms, and the larger is the total bubble on stocks.

A smaller public debt to GDP implies lower lump-sum transfers, which reduces publicly provided insurance. A higher price of new firms implies that entrepreneurs making an IPO become richer. Hence, comparing across stationary equilibria, agents with low savings are worse off the smaller the public debt and the larger the price bubble on equity, while the opposite is true for wealthy agents. From an ex-ante perspective and for a given real interest rate, average welfare comparing across steady states is thus higher the higher is public debt.\(^{14}\) For example, ex ante steady state average welfare is 0.44 percent higher in the benchmark stationary equilibrium where the entire bubble is on public debt than in an equilibrium with a public debt to GDP

\(^{13}\)Welfare is measured using a utilitarian social welfare function and expressed in terms of permanent changes in consumption.

\(^{14}\)In a more general model, there could be offsetting benefits from stock price bubbles. For example, investment might depend on the net worth of entrepreneurs or top managers.
Many observers believe that the recent financial crisis, as well as Great Depression and several other crises, were caused by collapsing asset price bubbles. Let us now illustrate how our model can capture such a crisis. In the equilibrium with a price bubble on equity corresponding to 26 percent of GDP, the stock market value to GDP ratio is 1.31.\(^{16}\) Suppose now that for some reason the price bubble on equity bursts, and suppose furthermore that policy is left unchanged. In the stock market crash, the value falls by 20 percent, back to the fundamental value of 1.05, as the bubble vanishes. As a consequence, the real value of government debt must increase from 0.95 to 1.2 as a fraction of GDP, reflecting the demand for assets that moves out of stocks and into bonds. This is our favored interpretation of the oft-repeated claim that “there is a scarcity of safe assets” during a financial crisis. Such an increase in the market value of government debt can come about in two very different ways. Either the authorities do nothing, and the economy experiences an immediate deflation of 19 percent.\(^{17}\) Alternatively, authorities accommodate the increased demand for bonds by immediately increasing the supply of nominal bonds.

Suppose first the authorities do nothing. If all prices are completely flexible, the surprise deflation associated with the bubble bursting has only little effect on real aggregate variables. The reason is that the fall in the value of stocks are exactly offset by an increase in the real value of public debt, which implies that each agent’s wealth is unchanged.\(^{18}\) The only effects come from an increase in the lump-sum transfer due to a larger public debt and a fall in the price of new public firms. These effects turn out to be quantitatively small.

However, there is widespread concern that deflation has severe consequences. To address this concern, let us now incorporate one channel by which deflation may affect real activity: We assume that nominal wages cannot not be adjusted downwards.\(^{19}\) More precisely, we impose the constraint \(w_t / p_{m,t} \geq w_{t-1} / p_{m,t-1}\) for all \(t \geq 0\). If prices fall, and nominal wages remain constant, real wages increase and firms’ labor demand fall. Since all agents of working-age supply one unit of labor inelastically, there is excess

\(^{15}\)Note that since lump-sum transfers and the price of new public firms differ across equilibria, the size of aggregate saving and thus the aggregate bubble (on private equity and on public debt) also differ across equilibria even though the real interest rate is the same.

\(^{16}\)To put this in perspective, the US stock market value to GDP ratio was 1.61 in 2000, and in the years leading up to the recent crisis, the ratio gradually increased to 1.42 in 2007. Source: see footnote 12.

\(^{17}\)As can be seen from equation (20) the implied deflation depends on our assumption of money growth, \(g^m\), technological change, \(g\), and the endogenous change in aggregate demand for government debt, \(g^x\). Here, demand increases by 26 percent.

\(^{18}\)Nominal government debt thus acts as a hedge against bubbles in the stock market. If the government had issued real one-period bonds instead, agents would not have been insured and rich agents would have experienced a significant loss of wealth.

\(^{19}\)There is a well known spike at zero in the nominal wage change distribution, and this is accentuated at low inflation rates. For recent international evidence and further references, see Holden and Wulfsberg (2014). For explanations of such rigidity, see Bewley (1999).
supply of labor in equilibrium. Suppose for simplicity that the fall in employment is shared equally by all workers. As long as the growth rate of nominal debt, $g^m$, is larger than real productivity growth, $g$, there is inflation in the stationary equilibrium (see equation (21)). In our benchmark calibration, inflation is 2 percent. Hence nominal stickiness will cease to bind at some period, after which the economy will converge to the same stationary equilibrium as if nominal wages were flexible.

Figure 1: Impact of crash on real variables

Figures 1 and 2 display the effect on real variables and financial prices following the burst of the bubble on equity for three different scenarios: (i) authorities do nothing and nominal wages are flexible, (ii) authorities do nothing and nominal wages are downward sticky, and (iii) authorities accommodate the increased demand for bonds. As just discussed, in scenario (i) there is an immediate deflation of 19 percent, but there is essentially no effect on real aggregate variables. With downward sticky nominal wages, the effect is very different. Deflation causes real wages to increase, causing a fall in labor demand. The resulting fall in labor earnings reduces the demand for all forms of assets, mitigating the deflationary pressure. In this scenario, there is an immediate deflation of 9 percent, and the real wage increases by 12 percent. This leads to a more than 40 percent drop in labor demand, and a 35 percent fall in output. The fall in labor demand causes the real interest to fall by 2.5 percentage points and become negative. The reduced demand for assets reduces investment; the physical capital stock gradually falls by almost 25 percent. Also, the value of the stock market falls more than
if wages were flexible; the immediate effect is 30 percent. That is, a significant decline in the fundamental value of the stocks adds to the initial 20 percentage points decline in the bubble value. The order of magnitude of the economic crisis under scenario (ii) thus resembles that of the Great Depression.

To understand what happens during the transition in scenario (ii), it is helpful to consider the evolution of government debt and of the real interest rate. As long as nominal stickiness binds, real wages are high, labor demand is depressed, and the real interest rate, \( \tilde{R}_t = \tilde{p}_t (1 + r_t^m) / \left[ \tilde{p}_t^m (1 + g^m) \right] \), is below that in the stationary equilibrium. The low interest rate causes the real value of debt to fall, which in turn implies that inflation is above its long-run level (see equation 20). Over time, the economy must converge to the ‘flexible wage’ stationary equilibrium. Hence, the real value of debt must increase as the real interest rate converges to the real interest in the stationary equilibrium, \( \tilde{R}_t = (1 + r_t^m) / (1 + g^m) \), from above. Thus, when nominal stickiness ceases to bind, the real interest jumps up. At the same time, labor demand and output jump up and the real wage and inflation jump down. From then on, all variables converge monotonically towards their stationary values. But as the figures show, even though nominal stickiness only binds for three periods (years) in his case, it has much more long-lasting effects on the economy. The welfare implications are also huge; the average welfare loss for agents in the economy is 7.8 percent.

The figures also show what happens if fiscal policy accommodates the increased
demand for bonds. The scenario we consider is one where accommodation occurs in the first period only. In all other periods policies follow the same path as before the crash. Furthermore, we assume that accommodation targets an inflation rate in the first period of 2 percent (as in the long-run). Even though this policy implies that the labor market immediately clears, it is quite different from doing nothing in a world of flexible wages. As discussed above, doing nothing under full flexibility implies that the wealth an agent has is unaffected; the fall in stock prices is exactly offset by an increase in the real value of debt. Under the accommodation scenario, the increase in the value of real debt enables massive transfers. To be precise, the government can transfer 22 percent of GDP in the first period. Of course, the agents will not consume such a windfall immediately, but save most of it. This scenario however effectively redistributes from wealthy agents with high saving rates to poorer agents with lower saving rates and compared to scenario (i) aggregate saving falls. This implies that the demand for bonds goes up by less, the value of the stock market falls more, and investment in physical capital decreases. The transition that follows can be understood in the same way as that under scenario (ii). Even though the outcome in terms of aggregate real variables is in between that of scenario (i) and (ii), the welfare implications are very different. Accommodating the increased demand for public debt avoids deflation and it enables large transfers. This results in an average welfare gain of 4.3 percent, as compared to a small gain of 0.49 percent under scenario (i) or a huge loss of 7.8 percent in scenario (ii).

Bad as it is, case (ii) is not the worst case scenario. Here, passive policy involves an unchanged rate of increase in nominal public debt. If the government were instead to respond to the crisis by reducing the rate of increase in nominal debt, it would take more time for real wages to fall into line and for employment to recover.

Even in the very long run, when nominal rigidities would presumably dissolve, a tight nominal debt ceiling could have severe consequences. For example, if the growth in nominal government debt is set to zero, the long-run real interest rate will be \( r^m + g \), which is above the original interest rate, and thus must entail a higher level of real debt.

The above logic shows that “nominal austerity” is bound to backfire. How, then, could authorities attain real austerity? If we define real austerity as a real ceiling for the public debt, the answer is plain: by committing to accelerate the issuance of nominal public debt. Permanently raising \( g^m \) entails higher inflation for a given nominal interest rate, and the reduction of the real interest rate serves to reduce the real value of public debt. Of course, the same objective could be attained through a permanent

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The average welfare gain could be even larger if the government aimed for higher inflation in the first period. For example, if the government issued debt targeting an inflation rate of 8 percent, the first period transfer would amount to 26 percent of GDP and the average welfare gain would be 5.2 percent.
reduction of the nominal interest rate. Whether this long-run outcome is desirable or not depends on the size of the original bubble. We know that it is not desirable for real interest rates to fall too far. Thus, if the original bubble was large, the new long-run outcome could also be significantly worse than the original equilibrium.

This example demonstrates the potency of expansionary debt policy, the dangers of nominally defined austerity, and the potential success of (indefinite) forward guidance. Could authorities instead have pursued a temporary expansionary interest rate policy with similar results? To probe this question, consider the effect of reducing the nominal interest rate from 3.5 percent to 0 percent in periods 1-4 (we assume that authorities cannot renge on the promise in period 0, i.e., when the bubble bursts). Call this scenario (iv). Strikingly, what appears to be quite expansionary monetary policy is largely undone via price effects: The capital stock, labor demand, and output falls a bit less than under passive policy, but there is still a massive crisis. The reduction in the nominal interest rate does however enable larger lump-sum transfers; see equation (19). During the four periods, the government can now distribute a lump-sum transfers of 4 percent of GDP, compared to 0.6 percent in the stationary equilibrium. While some of this transfer is saved, which explains why the capital stocks falls a bit less than under passive policy, the main benefit is the redistribution towards poor agents, who suffer disproportionately from the income loss associated with the decline in labor demand. Increased consumption among these agents implies that the ex ante average welfare loss is halved (to 3.9 percent).

If scenario (ii) resembles the Great Depression, the crisis of 2008 is more akin to scenario (iv), but with considerable fiscal policy added, both through automatic stabilizers and discretionary spending. Still, from 2008 onwards, consumer prices have increased much less than the pre-crisis trend, and we have had significant increases in unemployment.

An additional intriguing part of crises management in recent years has been the use of so-called quantitative easing, where new public debt is being created either to purchase old debt (shorten the maturity of public debt) or to buy private debt and equity. These policies too can be analyzed with the help of the present model. One equilibrium, which is in line with the fiscal theory of the price level, involves no change to inflation or any real variables. The asset purchase merely temporarily reallocates assets across private and public balance sheets.

21Krugman (1998) argues that such an extreme version of “forward guidance” of interest rates might not be credible. To address this concern, we would need to consider the political economy of monetary and fiscal policy. If unemployment is unevenly distributed, it is conceivable that the lucky majority would oppose welfare-maximizing policies.

22We cannot rule out that there are other equilibria, but we have been unable to find any.
6 Final Remarks

Within an otherwise standard modern macro-model, we have demonstrated that realistic market incompleteness in combination with realistic uninsurable income processes suffice to rationalize the low risk-free interest rates that we have seen over long periods in the past. Hence, our model provides a laboratory for the study of rational bubbles and Ponzi-values.

We find that the bursting of a rational bubble does not have to entail dire consequences, even if policy is unresponsive. If the bubble moves to another private asset, there will primarily be a shifting of wealth. If demand for saving moves from a bubble into real public debt, there will be low interest rates. However, if instead it moves into nominal public debt, the price level will go down – unless public debt is increased aggressively. In the presence of nominal contracting, such deflation can have dreadful implications.

We see several potential avenues for future work. One avenue is empirical: Does our model actually help to explain the vast heterogeneity in macroeconomic outcomes following asset market crashes? For example, are depressions primarily triggered when the bond markets are involved and there is a drop in the price level? Does it really matter for the effects of fiscal policy whether public debt is nominal or real? Can the model be calibrated to fit the detailed patterns of historical events?

Other avenues are theoretical. Our model appears to suggest that it is always better to let non-fundamental values take the form of public debt than private bubbles. Since many will be reluctant to believe in the feasibility and desirability of large public debt, it is important to investigate the counterarguments. Will the temptation to default on a larger debt be irresistible under more realistic circumstances? Will the promise of rescue operations have undesirable incentive effects? A preliminary answer to the last question is that a bubble will be less likely to burst if investors expect expansionary responses, since the loss from not moving quickly enough into government bonds will be smaller – but the smaller likelihood of bursting in turn might encourage the bubble to form.

Many other theoretical extensions appear straightforward. For example, it would be conceptually easy to increase realism by introducing more assets, such as land and housing, to introduce nominal price rigidities, elastic labor supply, and distortionary taxation. Likewise, it seems quite feasible to extend the model to allow for financial frictions or to have several forms of public debt with different properties as means of

\[23\]Within an OLG model that builds on Tirole (1985), Galí (2014) discusses whether policy-makers should raise interest rates in order to dampen fluctuations in an aggregate bubble. Galí argues that the answer is negative, as the bubble will fluctuate more when interest rates are higher. Our current analysis does not study anticipated fluctuations of the aggregate bubble, but it is obviously consistent with Galí’s view that the bubble grows faster when interest rates are higher.
payment – currency in addition to bonds.

In our working paper, Domeij and Ellingsen (2014), we explore the model’s comparative static properties. Specifically, we characterize the long-run trade-off between output and real interest rates and optimal interest rate policy provided authorities are free to choose any equilibrium\(^\text{24}\). We also investigate how policy should respond to permanent changes in the growth rate and the technology turnover rate\(^\text{25}\).

A more ambitious extension would be to introduce private financial intermediation. We conjecture that a realistic model of intermediation implies that intermediaries earn Ponzi-value, and a natural objective would be to assess the size and the social costs and benefits of non-fundamental values lodged in the financial sector.

To the extent that such private intermediation involves nominal contracts, deflation obviously has the potential to create havoc in debt markets. Pursuing this extension might thus admit a reformulation of Fisher’s (1933) debt-deflation hypothesis.

On the more conceptual side, a shortcoming of our model is that we only consider totally unexpected movements from one equilibrium to another. One way to address this objection would be to introduce an explicit “sunspot” process. Then, the current analysis would presumably emerge as a limiting case. Another, more ambitious, extension is to introduce equilibrium refinements. Our current approach of considering all rational-expectations equilibria, and allowing authorities choose (more or less freely) one equilibrium in this set, may be overlooking other coordination mechanisms. For example, is it possible that coalitions of agents will find it easier or more attractive to introduce private bubbles in case interest rates become low? And just how close to the growth rate can the real interest rate go before investors in private bubbles get nervous? More generally, as for any model that admits multiple equilibria, it is desirable to understand when and how “animal spirits” are likely to set in. While problems of equilibrium selection are well known to be a challenging, there has been progress. For example, Morris and Shin (1998) provide a theory of equilibrium selection based on strategic risk (endogenous uncertainty) in economies with infinitely many agents. A great virtue of such an extension is that it would give rise to bubble risk premia in our setting.

Aggregate uncertainty, exogenous or endogenous, is a desirable feature not least because it might generate additional asset return differentials. Studying exogenous aggregate income shocks, Krusell, Mukoyama, and Smith (2013) derive the magnitude of such return differentials, and show that they might be quite large, in a pure endowment economy without durable assets and with tight borrowing constraints. However, absent bubbles, we conjecture that risk premia deriving from exogenous

\(^{24}\) As emphasized by Dávila et al (2012), \( r < g \) does not imply dynamic inefficiency in a dynastic model, unlike the OLG-model of Diamond (1965).

\(^{25}\) The working paper also contains some sensitivity analysis, a comprehensive analysis of a pure endowment model, as well as a more detailed discussion of related literature.
shocks would be quite small in our setting, which has extensive opportunities for saving in durable assets. On the other hand, with endogenous (i.e., bubble) risk, the risk premia could well be large. Only in the special case with full price and wage flexibility will government bonds serve as a perfect hedge in case a bubble moves from stocks to bonds. In the other cases we study, rich savers will lose when the bubble migrates to the bond market – either because of a depression caused by deflation or because authorities expand the supply of bonds in order to prevent such deflation. Indeed, we see undiversifiable bubble risk as being a natural candidate for some of the “rare disasters in stock markets that have been invoked to explain the equity premium and excess volatility puzzles (Rietz, 1988).

Bubble risk will naturally be largest for those assets whose bubble is larger relative to their fundamental value. In the stock market, these are the “value” stocks rather than the “growth” stocks. We thus think that analysis along these lines might contribute an explanation not only for the excess volatility puzzle and the equity premium puzzle, but also for the value premium.

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